

## Pengamiran

Pengamiran boleh dilihat sebagai proses songsang untuk

Jika beza  $f(x)$  memberikan  $g(x)$ ,

Maka,

$$\frac{d}{dx} f(x) = g(x) \leftrightarrow$$

$$\frac{d}{dx} \quad \rightarrow \text{beza terhadap}$$

$$\int \dots dx \rightarrow$$

$\rightarrow$  Mesti tulis  $\int \dots dx$

$$y = 3x$$

$$y = 3x + 1$$

$$y = 3x - 4 \quad \begin{array}{l} \rightarrow \\ \leftarrow \end{array} \quad \frac{dy}{dx} =$$

$$y = 3x + a^2$$

$$\therefore \int 3 dx =$$

$$\frac{d}{dx} f(x) = g(x) \leftrightarrow$$

$c$  ialah pemalar pengamiran

$$y = -\frac{1}{x} \cdot \text{Cari } \frac{dy}{dx} \cdot \text{Maka, cari } \int \frac{1}{x^2} dx$$

$y = x^2 \quad \frac{dy}{dx} = 2x$	$\int$
$y = \frac{x^3}{3} \quad \frac{dy}{dx} = x^2$	
$y = x^4 + 3x - 7$	
$\frac{dy}{dx} =$	

$$\frac{d}{dx}(kx) = \quad \int$$

$$\int 5 dx =$$

$$\int -\frac{1}{7} dx =$$

## Pembezaan

$$\frac{d}{dx} k[f(x)] = k \frac{d}{dx} [f(x)]$$

$$\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)]$$

$\rightarrow$  kalau darab pemalar, pemalar

$\rightarrow$  kalau tambah/tolak sebutan,

## Pengamiran

$$\int k[f(x)] dx =$$

$$\int [f(x) \pm g(x)] dx =$$

$\rightarrow$  kalau darab pemalar, pemalar

$\rightarrow$  kalau tambah/tolak sebutan

Jika  $\frac{d}{dx} f(x) = g(x)$ , cari

i)  $\int g(x) dx =$

ii)  $\int 3g(x) dx =$

iii)  $\int [g(x) - 3] dx =$

Pemalar  $c$  tidak perlu dilibatkan dalam operasi algebra lain kerana mewakili sebarang nilai

Jika  $\frac{d}{dx} f(x) = 2g(x)$ , cari

i)  $\int 2g(x)dx =$

ii)  $\int g(x)dx =$

iii)  $\int 5g(x)dx =$

$y = x^3$ . Cari  $\frac{dy}{dx}$ . Maka, cari  $\int 2x^2 dx$

Semak :

Kadangkala perlu sengaja masukkan pemalar

$$\int f(x)dx =$$

Jawapan pengamiran boleh disemak dengan membezakan jawapan untuk mendapatkan balik soalan

$$\frac{d}{dx}(x^n) =$$

→

→

$$\int x^n dx =$$

→

→

a)  $\int x dx =$

b)  $\int x^3 dx =$

c)  $\int 4x^2 dx =$

d)  $\int \frac{5}{x^2} dx =$

e)  $\int (6x^2 - 3x + 2) dx$

=

Pastikan jelas langkah pengamiran VS langkah penukaran / permudahkan

Bandingkan

$$\frac{d}{dx}(3) =$$

$$\int 3 dx =$$

$$\frac{d}{dx}(x) =$$

$$\int x dx =$$

$$\frac{d}{dx}(3x) =$$

$$\int 3x dx =$$

$$\frac{d}{dx}(x+3) =$$

$$\int (x+3) dx =$$

$$\frac{d}{dx}(x^2) =$$

$$\int x^2 dx =$$

$$\frac{d}{dx}(ax+b)^n =$$

→

→

→

→

$$\int (ax+b)^n dx =$$

→

→

→

→

Bandingkan

$$\frac{d}{dx}(2x-7)^4 \qquad \int (2x-7)^4 dx$$

=

$$\text{a) } \int (3x+2)^2 dx =$$

$$\text{b) } \int 5(x+3)^6 dx =$$

$$\text{c) } \int (1-x)^4 dx =$$

$$\text{d) } \int \frac{5}{(7-2x)^2} dx =$$

Bandingkan

$$\int (x+1)^2 dx =$$

$$\int (x^2+1) dx =$$

$$\int x^n dx \quad \rightarrow \quad x \text{ kuasa } n$$

$$\int (ax+b)^n dx \quad \rightarrow$$

$$\text{a) } \int (x^2-1)^2 dx =$$

$$\text{BUKAN } \int (x^2-1)^2 dx = \frac{(x^2-1)^3}{3(2x)} + c$$

$$\int [f(x)]^n dx$$

→ Tiada formula mudah jika

$$\int [f(x)g(x)] dx, \int \frac{f(x)}{g(x)} dx$$

→ Tiada formula mudah

$$\text{b) } \int x(x^2-1) dx =$$

$$\text{c) } \int \frac{x^4-5x^2}{x} dx =$$

Kadangkala perlu kembang /  
pisahkan pecahan SEBELUM pengamiran

Bandingkan

$$\int (x+1)dx \rightarrow$$

$$\int x(x+1)dx \rightarrow$$

$$\int (x+1)^2 dx \rightarrow$$

$$\int (x^2+1)dx \rightarrow$$

$$\int (x^2+1)^2 dx \rightarrow$$

$$\int (x+1)(x-2)dx \rightarrow$$

$$\int 2(x+1)^2 dx \rightarrow$$

$$\int 2(x+1)dx \rightarrow$$

$$\int 2x(x+1)dx \rightarrow$$

$$\int 5(3x-2)^2 dx \rightarrow$$

$$\int x(3x-2)^2 dx \rightarrow$$

$$\int 5x(3x-2)^2 dx \rightarrow$$

$$\int \frac{(3x-2)^2}{4} dx \rightarrow$$

$$\int \frac{3x^2-2x}{x} dx \rightarrow$$

### Fungsi kecerunan

$$y = \dots \qquad \frac{dy}{dx} = \dots$$

pers. lengkung                      fungsi kecerunan

Suatu lengkung mempunyai fungsi  
kecerunan  $3x-4$ , dan melalui titik  $(2,7)$ .  
Cari persamaan lengkung tersebut.

$$\text{fungsi kecerunan} \qquad \text{pers. lengkung}$$
$$\frac{dy}{dx} = \dots \qquad y =$$

### Kamiran Tentu

$$\text{Jika } \int f(x)dx = g(x) + c$$

$$\int_a^b f(x)dx =$$

→ Jawapan kamiran tentu ialah

Mengapakah tidak perlu tulis  $c$  ?

$$\int_a^b f(x)dx = [g(x) + c]_a^b$$

$$\text{a) } \int_1^3 2x dx =$$

Nilai kamiran tentu boleh di SEMAK dengan  
kalkulator

### Kamiran Tentu

→  
→  
→

$$b) \int_{-1}^1 (x^3 + 1) dx =$$

$$c) \int_0^4 (2x + 1)^3 dx =$$

$$[kg(x)]_a^b =$$

$$[f(x) \pm g(x)]_a^b =$$

→ boleh keluarkan pemalar

→ boleh pisahkan sebutan

Bandingkan

$$\int x^3 dx =$$

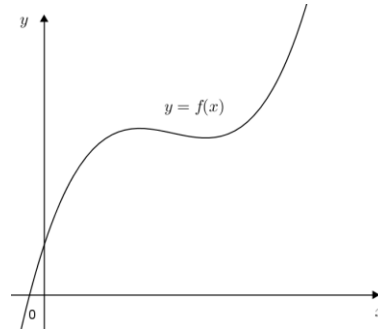
$$\int_0^1 x^3 dx =$$

Kamiran tentu  $\int_a^b f(x) dx$  sebenarnya

mewakili

→ luas di bawah graf

→ dari



Katakan  $\int f(x) dx = g(x) + c$

$$i) \int_a^a f(x) dx =$$

$$\int_a^a f(x) dx \rightarrow \text{luas dari}$$

→

$$ii) \int_a^b f(x) dx =$$

$$\int_b^a f(x) dx =$$

∴

$$iii) \int_a^b f(x) dx + \int_b^c f(x) dx \setminus$$

$$=$$

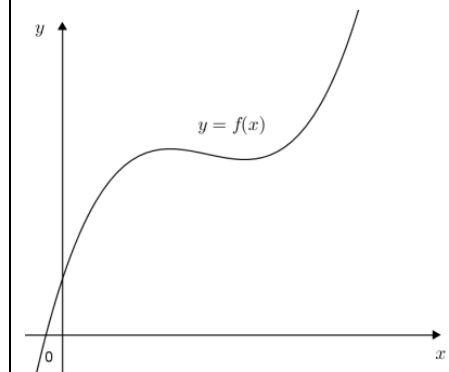
$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$



luas

+ luas

=



$$iv) \int_a^b kf(x) dx =$$

$$\int_a^a f(x) dx =$$

$$\int_b^a f(x) dx \quad \int_a^b f(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx =$$

$$\int_a^b kf(x) dx =$$

Diberi  $\int_1^4 f(x) dx = 4$  . Cari

a)  $\int_4^1 f(x) dx =$

b)  $\int_1^4 [4f(x) - 3] dx =$

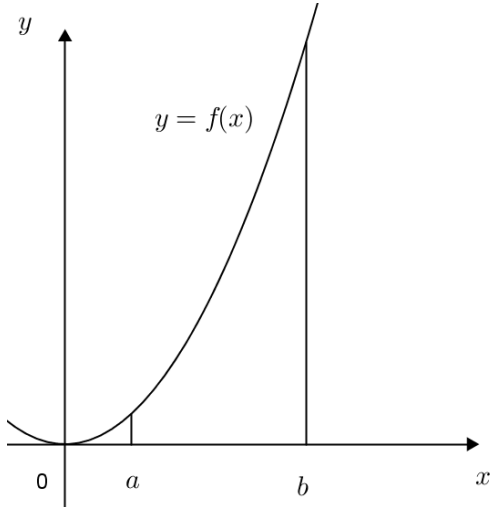
c) Diberi  $\int_1^4 [kf(x) - 4x] dx = 54$  . Cari nilai  $k$

Bandingkan

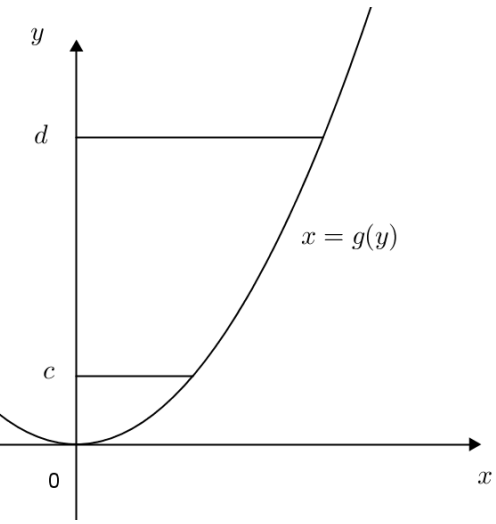
$$\int_a^b kf(x) dx =$$

$$\int_a^b [f(x) + k] dx =$$

Definisi sebenar

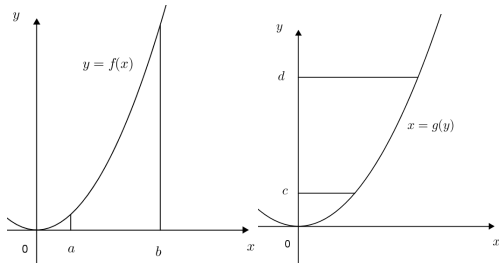
$$\int_a^b y dx = \lim_{\delta x \rightarrow 0} \sum y \delta x$$


$y = f(x)$

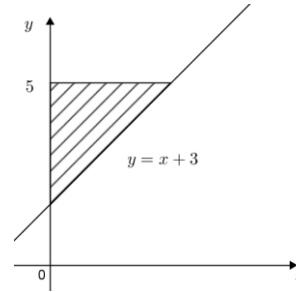
$$\int_c^d x dy = \lim_{\delta y \rightarrow 0} \sum x \delta y$$


$x = g(y)$

Bandingkan



Cari luas kawasan berlorek



Bandingkan

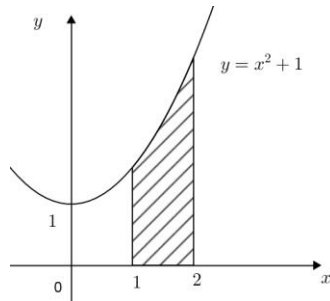
$$\int x \, dx =$$

$$\int y \, dx \rightarrow$$

$$\int y \, dy =$$

$$\int x \, dy \rightarrow$$

Cari luas kawasan berlorek



Bandingkan

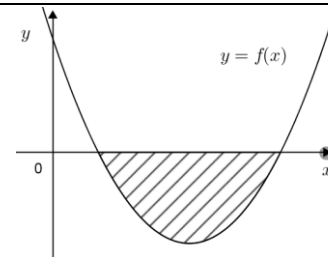
$$\int_a^b y \, dx$$

$$\int_c^d x \, dy$$

Perhatikan

$$\int (x+1) \, dx =$$

$$\int (y+1) \, dy =$$



$$\int_a^b y \, dx = \lim_{\delta x \rightarrow 0} \sum y \delta x$$

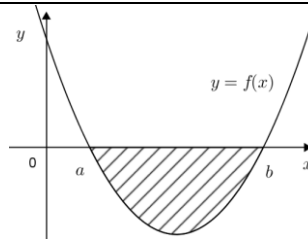
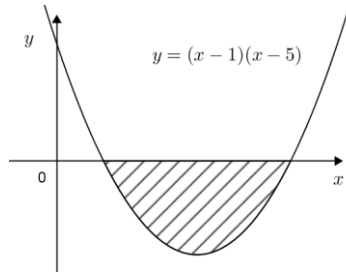
Jadi jika graf di bawah paksi - x ,

→ y akan bernilai

→  $\int_a^b y \, dx$  akan bernilai

→ tetapi kita menganggap luas adalah

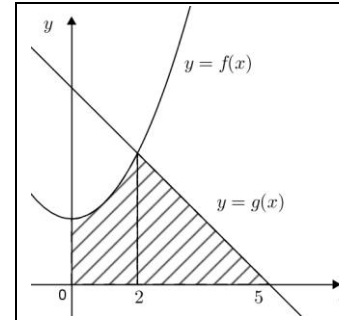
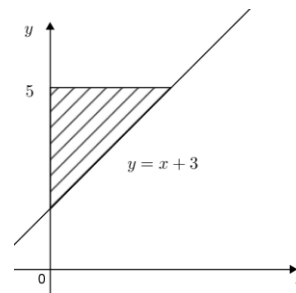
Cari luas kawasan berlorek



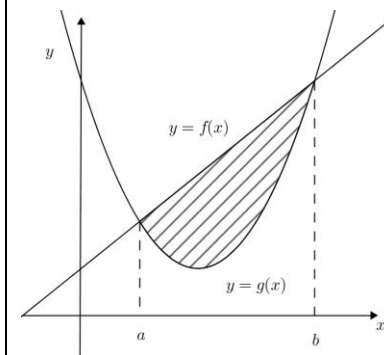
Diberi luas kawasan berlorek ialah 5 unit<sup>2</sup>

Jadi  $\int_a^b f(x) dx =$

Kita juga boleh guna formula-formula luas untuk bentuk geometri tertentu



Luas =

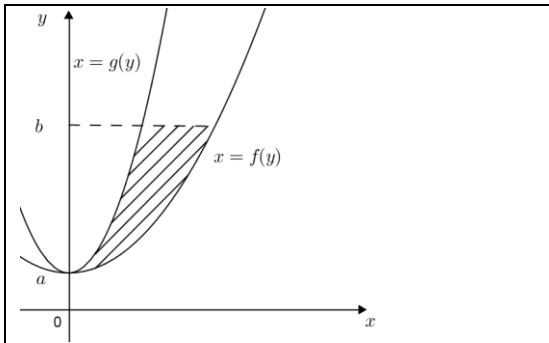


Luas =

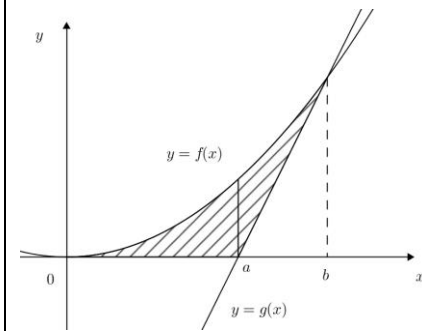
Jika luas dibawah paksi - x

$\rightarrow \int_a^b y dx$                        $\rightarrow$  Luas



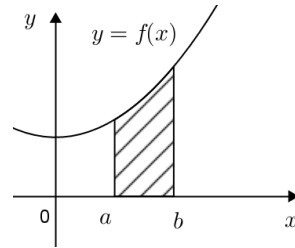


Luas =

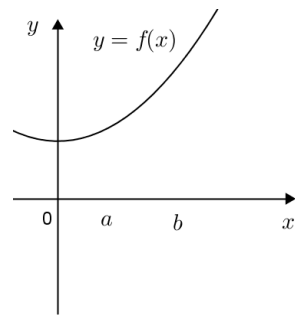


Luas =

**Isipadu kisanan**



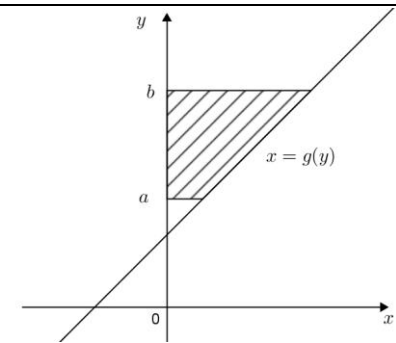
↓  
Dikisarkan 360°  
melalui paksi- x



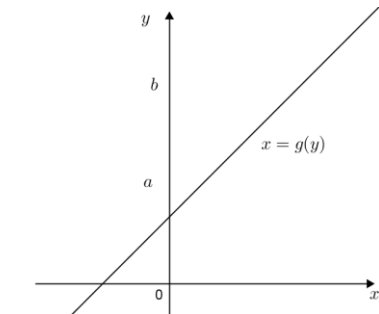
Bongkah boleh dipotong menjadi banyak

→setiap mempunyai isipadu

Isipadu kisanan =  
(melalui paksi- x )



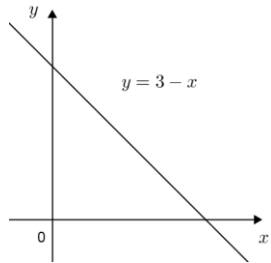
↓  
Dikisarkan 360°  
melalui paksi- y



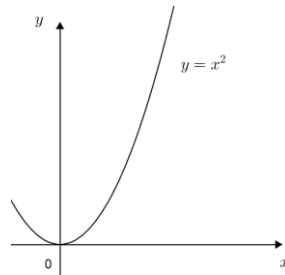
→setiap mempunyai isipadu

Isipadu kisanan =  
(melalui paksi- y )

Cari isipadu kisanan bila kawasan yang dibatasi garis lurus, paksi-  $y$  dan paksi-  $x$  dikisarkan  $360^\circ$  melalui paksi-  $x$



Cari isipadu kisanan bila kawasan yang dibatasi lengkung, paksi-  $y$  , dan garis  $y = 3$  dikisarkan  $360^\circ$  melalui paksi-  $y$



Bandingkan

$$\int x^2 dx = \qquad \int y^2 dx \rightarrow$$

$$\int y^2 dy = \qquad \int x^2 dy \rightarrow$$

Jika  $y = x + 1$

$$y^2 \rightarrow$$

$$x \rightarrow$$

$$x^2 \rightarrow$$

Jika  $y = x^2 + 1$

$$x^2 \rightarrow$$

$$y^2 \rightarrow$$

$$x \rightarrow$$